INTEGRATION BY PARTS

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Name: Solutions

Evaluate the following integrals

1. $\int x\cos(x)\,\mathrm{d}x$

$u = x \qquad v = \sin(x)$

$du = dx \qquad dv = \cos(x)dx$

$$\int x\cos(x)dx = x\sin(x) - \int \sin(x)dx$$

$$= \boxed{x\sin(x) + \cos(x) + C}$$

2. $\int \ln(x)^2\,\mathrm{d}x$

$u = \ln(x)^2 \qquad v = x$

$du = \frac{2\ln(x)}{x}dx \qquad dv = dx$

$u = \ln(x) \qquad v = x$

$du = \frac{1}{x}dx \qquad dv = dx$

$$\int \ln(x)^2 dx = x\ln(x)^2 - \int x\frac{2\ln(x)}{x}dx$$

$$= x\ln(x)^2 - 2\int \ln(x)dx$$

$$= x\ln(x)^2 - 2\left(x\ln(x) - \int x\frac{1}{x}dx\right)$$

$$= \boxed{x\ln(x)^2 - 2\ln(x) + 2x + C.}$$

3. $\int e^{2\theta}\sin(3\theta)\,\mathrm{d}\theta$

$u = \sin(3\theta) \qquad v = \frac{1}{2}e^{2\theta}$

$du = 3\cos(3\theta) \qquad dv = e^{2\theta}d\theta$

$$\int e^{2\theta}\sin(3\theta)d\theta = \frac{1}{2}e^{2\theta}\sin(3\theta) - \frac{3}{2}\int e^{2\theta}\cos(\theta)d\theta$$

$u = \cos(3\theta) \qquad v = \frac{1}{2}e^{2\theta}$

$du = -3\sin(3\theta) \qquad dv = e^{2\theta}d\theta$

$$= \frac{1}{2}e^{2\theta}\sin(3\theta) - \frac{3}{2}\left(\frac{1}{2}e^{2\theta}\cos(3\theta) + \frac{3}{2}\int e^{2\theta}\sin(3\theta)d\theta\right)$$

$$= \frac{1}{2}e^{2\theta}\sin(3\theta) - \frac{3}{4}e^{2\theta}\cos(3\theta) - \frac{9}{4}\int e^{2\theta}\sin(3\theta)d\theta$$

$$\Rightarrow \left(1+\frac{9}{4}\right)\int e^{2\theta}\sin(3\theta)d\theta = \frac{1}{2}e^{2\theta}\sin(3\theta) - \frac{3}{4}e^{2\theta}\cos(3\theta) + C$$

$$\Rightarrow \boxed{\int e^{2\theta}\sin(3\theta)d\theta = \frac{2}{13}e^{2\theta}\sin(3\theta) - \frac{3}{13}e^{2\theta}\cos(3\theta) + C.}$$

4. $\int (x^2+1)e^{-1}\,\mathrm{d}x$

$$\int (x^2+1)e^{-1}dx = e^{-1}\int (x^2+1)dx$$

$$= \boxed{\frac{1}{e}\left(\frac{1}{3}x^3 + x\right) + C.}$$

5. $\int \arctan(\theta)\,\mathrm{d}\theta$

$u = \arctan(\theta) \qquad v = \theta$

$du = \frac{1}{1+\theta^2}d\theta \qquad dv = d\theta$

$$\int \arctan(\theta)d\theta = \theta\arctan(\theta) - \int \theta\left(\frac{1}{1+\theta^2}\right)d\theta$$

$u = 1+\theta^2$

$du = 2\theta d\theta$

$\Rightarrow \frac{1}{2}du = \theta d\theta$

$$= \theta\arctan(\theta) - \int \frac{1}{2u}du$$

$$= \theta\arctan(\theta) - \frac{1}{2}\ln|u| + C$$

$$= \boxed{\theta\arctan(\theta) - \frac{1}{2}\ln(1+\theta^2) + C.}$$