Parametric Equations and Polar Coordinates

10.5 Conic Sections

B. Farman

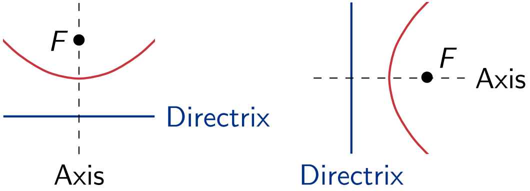
Mathematics and Statistics
Louisiana Tech University

Calculus III

Parabolas

Definition (Parabolas)

A **parabola** is the set of points in a plane that are equidistant from a foxed point, F, called the **focus** and a fixed line, called the **directrix**. The point halfway between the focus and the directrix is called the **vertex**. The line through the focus perpendicular to the directrix is called the **axis** of the parabola.



Parabolas

Equations of Parabolas

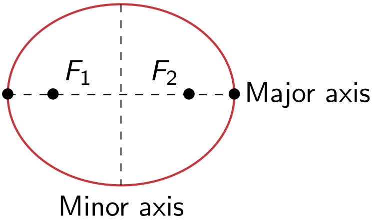
An equation of the parabola with

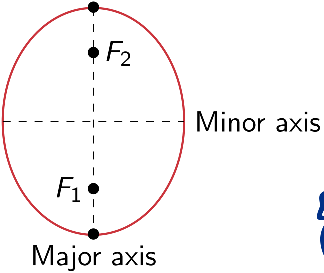
1. focus $(0, p)$ and directrix $y = -p$ is $x^2 = 4py$.
2. focus $(p, 0)$ and directrix $x = -p$ is $y^2 = 4px$.

Ellipses

Definition (Ellipse)

An **ellipse** is the set of points in a plane the sum of whose distances from two **foci**, F_1 and F_2, is constant. The line segment through the ellipse and the foci is called the **major axis**, while the perpendicular line segment through the ellipse is called the **minor axis**. The endpoints of the major axis are called the **vertices**.





Ellipses

Equation of an Ellipse

Assume $a \geq b > 0$ are constants. The ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

has foci $(\pm\sqrt{a^2 - b^2}, 0)$, and vertices $(\pm a, 0)$.

Ellipses

Equation of an Ellipse

Assume $a \geq b > 0$ are constants. The ellipse

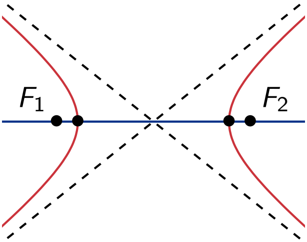
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

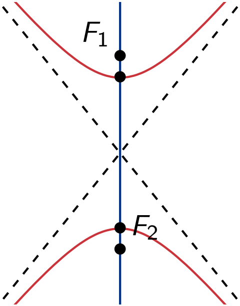
has foci $(0, \pm\sqrt{a^2 - b^1}$, and vertices $(0, \pm a)$.

Hyperbolas

Definition

A **hyperbola** is the set of all points in a plane the difference of whose distances from two **foci**, F_1 and F_2, is a constant. The hyperbola is divided into two **branches** with diagonally opposite sides tending to the same line called an **asymptote**. Each branch intersects the line segment between the foci at a **vertex**.





Hyperbolas

Equation of a Hyperbola

The hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

as foci $(\pm\sqrt{a^2 + b^2}, 0)$, vertices $(\pm a, 0)$, and asymptotes $y = \pm(b/a)x$.

Hyperbolas

Equation of a Hyperbola

The hyperbola

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

as foci $(0, \pm\sqrt{a^2 + b^2})$, vertices $(0, \pm a)$, and asymptotes $y = \pm(a/b)x$.